$$x+m_1y=x_1+m_1y_1,\ x+m_2y=x_2+m_2y_2.$$

It is proposed to find the coördinates of the intersection of these lines when (x_1, y_1) approaches and becomes coincident with (x_2, y_2). Eliminating x between the equations, we have

$$y=y_1+\frac{1+m_2\left(\dfrac{y_1-y_2}{x_1-x_2}\right)}{\dfrac{m_1-m_2}{x_1-x_2}}.$$

Since $\underset{x_1=x_2}{\text{Lim.}}\left(\dfrac{y_1-y_2}{x_1-x_2}\right)=\dfrac{dy_1}{dx_1}$ and $\underset{x_1=x_2}{\text{Lim.}}\left(\dfrac{m_1-m_2}{x_1-x_2}\right)=\dfrac{d^2y_1}{dx_1^2}$, we have

$$y=y_1+\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y}{dx^2}},\ \text{and } x=x_1-\frac{dy_1}{dx_1}\left\{\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y_1}{dx_1^2}}\right\}.$$

105. Proposed by **CHARLES C. CROSS**, Meridithville, Va.

From all points in a straight line passing through the center of a given circle tangents are drawn to the circle. If the bases and vertices of all the angles thus formed are made to coincide; required the equation of the curve passing through the tangent points.

Solution by **J. W. YOUNG**, Oliver Graduate Scholar in Mathematics, Cornell University, Ithica, N. Y.

Let the circle be $x^2+y^2=a^2$, and the given line $y=0$. Then the length of the tangent from any point $(x_1, 0)$ on the given line is $\sqrt{(x_1^2-a^2)}$.

Also the slope of the tangent may be calculated from the equation $mx_1\pm a\sqrt{(1+m^2)}=0$, which is obtained by substituting the point $(x_1, 0)$ in the tangent equation $y=mx\pm a\sqrt{(1+m^2)}$. Solving for m, we have

$$m^2=\frac{a^2}{x_1^2-a^2},\ \text{or } m=\pm\frac{a}{\sqrt{(x_1^2-a^2)}}.$$

To determine the required locus, use polar coördinates, with the common vertex of angles as pole and their common base as initial line; the coördinates (r, θ) of the tangent points are then given by the equations

$$\left.\begin{aligned} r&=\sqrt{(x_1^2-a^2)} \\ \tan\theta&=\frac{\pm a}{\sqrt{(x_1^2-a^2)}} \end{aligned}\right\}$$

Hence, the required locus is $r\tan\theta=\pm a$; in Cartesian coördinates this becomes $x^2(y^2-a^2)+y^4=0$.

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.